Kolmogorov Entropy and Physics Beyond the Standard Model (1)

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Abstract

We argue here that the high-energy behavior of fundamental interactions can be interpreted as manifestation of Kolmogorov (-K) entropy. The conventional classification of fields based on Poincaré symmetry appears to be rooted in the chaotic regime of nonlinear dynamics far above the Standard Model scale.

Key words: Geodesic flows, Riemannian manifolds, Hamiltonian chaos, Kolmogorov entropy, multifractals, information dimension, Physics Beyond the Standard Model.

1. Introduction

Ergodic behavior occurs in a variety of physical contexts. For example, in isolated systems, ergodicity reflects the onset of thermodynamic equilibrium, whereby all the accessible microstates of the phase-space are equiprobable over a sufficiently long period of time [1-2].

It is known that Hamiltonian systems are universal models of classical and quantum physics. A remarkable property of these systems is that they can be described as geodesic flows on Riemannian manifolds [3-5]. It is also known that the ergodic regime of Hamiltonian dynamics arises from the divergence of geodesics on manifolds with negative curvature (so-called *hyperbolic manifolds*), which drive the transition from integrability to chaos [3-8]. Research shows that Hamiltonian chaos is not restricted to hyperbolic manifolds, but that it extends to large N-body systems analogous to fluid flows ($N \to \infty$) and to systems displaying parametric instability [3-5].

Although many aspects of chaos in classical field theory remain to be settled, several facts are well understood. For instance, it is known that,

a) sensitivity of geodesics to initial conditions is a universal marker of the transition from integrability to chaos.

b) natural signatures of chaotic behavior include topological mixing and the existence of a non-zero K-entropy.

c) The N-body problem of Newtonian gravitation ($N > 2$) and the dynamics of non-abelian gauge fields have non-vanishing K-entropies and exhibit chaotic behavior [5, 8].

d) far above the Fermi scale of particle physics, open quantum systems are likely to undergo *decoherence*, which is triggered by entanglement with an ever-fluctuating environment. As irreversible loss of phase information, decoherence causes the transition from quantum to classical behavior and favors the onset of Hamiltonian chaos.

In view of a)-d), the goal of this work is to explore whether the high-energy sector of fundamental interactions may be interpreted as manifestation of K-entropy. We base our approach on a two-fold rationale:

1) The high energy regime of field theory falls outside thermodynamic equilibrium and is manifestly *unstable*,

2) Dynamical instability generated by out-of-equilibrium conditions is *universal* and, as such, it underlies both Hamiltonian chaos and multi-body gravitational physics. Critical phenomena of statistical physics support this assumption, based on (at least) two premises [9-11]:

2.1) The universal route to chaos of dissipative nonlinear maps,

2.2) Universal critical exponents of second-order phase transitions.

This contribution is divided into two parts, and follows a step-by-step approach, with emphasis on concision and clarity. The *first part* includes section 2, which briefly surveys the geometrization of Hamiltonian dynamics. In the *second part* of the paper, sections 3 to 4 bridge the gap between the evolution of classical fields, K-entropy, and its corresponding information dimension. Section 5 reveals the current limitations of Hamiltonian chaos in dealing with fermion fields. Section 6 builds upon the conjecture that Hamiltonian chaos lies behind the conventional classification of fields based on the Poincaré symmetry. Concluding remarks are summarized in section 7.

The reader is cautioned upfront on the preliminary nature of this investigation, which requires independent validation and further consolidation of ideas.

2. Geometrization of Hamiltonian dynamics

A conservative system of classical fields is defined by the Hamiltonian [3-4]

$$H = \frac{1}{2} a_{ij} \dot{\varphi}_i \dot{\varphi}_j + V(\varphi_1, \varphi_2, ..., \varphi_N) \tag{1}$$

where $H = E$ is a constant of motion. The configuration space M of the system consists of N local coordinates $(\varphi_1, \varphi_2, ..., \varphi_N)$ and can be associated with a Riemannian metric using the substitution

$$\boxed{g_{ij} = 2\left[E - V(\varphi)\right] a_{ij}} \tag{2}$$

The minimum action principle reads

$$\delta I = \delta [\int_{\gamma(t)} \frac{\partial L}{\partial \dot{\varphi}_i} \dot{\varphi}_i \, dt] = 0 \tag{3}$$

where integration is performed along the path $\gamma(t)$. The kinetic energy of the system takes the form

$$W=\frac{1}{2}\dot{\varphi}_i\frac{\partial L}{\partial\dot{\varphi}_i}=E-V(\varphi) \tag{4}$$

(3) becomes, accordingly

$$\delta\int_{\gamma(t)}2Wdt=\delta\int_{\gamma(t)}\sqrt{g_{ij}\dot{\varphi}_i\dot{\varphi}_j}\,dt=\delta\int_{\gamma(s)}ds=0 \tag{5}$$

It follows from (5) that natural motions of the Hamiltonian system are geodesics of M, whose differential arclength is ds. The corresponding geodesic equation is given by

$$\frac{d^2\varphi^i}{ds^2}+\Gamma^i_{jk}\frac{d\varphi^j}{ds}\frac{d\varphi^k}{ds}=0 \tag{6}$$

in which Γ^i_{jk} stand for the Christoffel coefficients of the metric. Let ξ^i denote the distance separating two adjacent geodesics on M. The *Jacobi equation* represents the evolution of ξ^i in the original time coordinate t and, for low-dimensional systems ($N=2$), can be presented as [4]

$$\frac{d^2\xi^i}{dt^2} + K_R\,\xi^i = 0 \tag{7}$$

in which K_R is the local curvature of the manifold. The local curvature of Hamiltonian systems described by (1) can be evaluated in closed form according to

$$K_R \equiv \Delta V = \sum_{i=1}^{N} \frac{\partial^2 V}{\partial \varphi_i^2} \tag{8}$$

Chaos unfolds when the separation of adjacent geodesics grows exponentially on hyperbolic manifolds ($K_R < 0$), as in

$$\xi^i(t) = \xi^i(0)\exp[\lambda^i(t)\cdot t] \tag{9}$$

where $\lambda^i(t) > 0$ are positive Lyapunov exponents. However, as mentioned in the Introduction, chaos can also develop from a fluctuating positive curvature K_R along the geodesics. This type of instability occurs when certain system parameters become time dependent. An elementary example of *parametric instability* is a harmonic oscillator acted upon a periodically modulated frequency. The mean and variance of curvature fluctuations of this unstable regime are, respectively,

$$\kappa_0 = \frac{\langle K_R \rangle}{N} \tag{10}$$

$$\sigma_\kappa^2 = \frac{\langle (K_R - \langle K_R \rangle)^2 \rangle}{N} \tag{11}$$

Assuming that the system is defined by a single maximal Lyapunov exponent (λ_{MAX}) and considering the limit $\sigma_\kappa << \kappa_0$, leads to the following approximation

$$\boxed{\lambda_{MAX} \propto \sigma_k^2} \tag{12}$$

As it will become apparent later, relations (8-12) can be used to study the *unstable sector* of field theories operating above the Standard Model scale. Here are few elementary examples of such theories:

a) Complex scalar field in a Higgs-like potential

$$V(\varphi_1, \varphi_2) = \mu^2(\varphi_1^2 + \varphi_2^2) + \lambda(\varphi_1^2 + \varphi_2^2)^2 \tag{13}$$

b) Newtonian gravity of multi-body systems

$$V(x) = -G \sum_{a<b} \frac{m_a m_b}{\left| x_a - x_b \right|} \tag{14}$$

c) Two-dimensional classical Yang-Mills theory [8]

$$H_{YM} = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2) + V(q_1, q_2) \tag{15}$$

$$V(q_1, q_2) = \frac{1}{2} g^2 q_1^2 q_2^2 \tag{16}$$

d) Scalar electrodynamics in four dimensional spacetime ($\mu = 0, 1, 2, 3$)

$$V(\varphi, A) = j_\mu A^\mu - \frac{e^2}{2} A_\mu A^\mu \varphi \varphi^* \tag{17}$$

where the scalar current is

$$j_\mu = -\frac{ie}{2}(\varphi^*_{,\mu}\varphi - \varphi_{,\mu}\varphi^*) \tag{18}$$

e) Fermion current coupled to the electromagnetic field

$$V(\psi, A) = e A_\mu \overline{\psi} \gamma^\mu \psi \tag{19}$$

There is a key caveat in using (19), in that (19) works in connection with the relativistic Lagrangian

$$L_D(\psi) = i\overline{\psi}\gamma^\mu \partial_\mu \psi + m\overline{\psi}\psi \tag{20}$$

which fails to be quadratic in field velocities, as demanded by (1). This issue will be revisited in the second part of the paper.

References (part 1)

1. Tolman, R. C., The Principles of Statistical Mechanics, Dover, 1979.

2. Landau L. D. and Lifshitz E. M., Statistical Physics, Pergamon Press, 1970.

3. https://arxiv.org/pdf/chao-dyn/9702011.pdf

4. https://arxiv.org/pdf/chao-dyn/9609010.pdf

5. https://arxiv.org/pdf/2202.09846.pdf

6. Gutzwiller M. C., Chaos in Classical and Quantum Mechanics, Springer-Verlag, 1990.

7. Zaslavsky G. M., Hamiltonian Chaos and Fractional Dynamics, Oxford Univ. Press, 2006.

8. Savvidy G. K., “The Yang-Mills Classical Mechanics as a Kolmogorov K-system”, Phys. Lett. 130B, No. 5 (1983).

9. Feigenbaum M. J., “The universal metric properties of nonlinear transformations”, J. Stat. Phys. 21:669–706 (1979).

10. Amit D. J and Mayor V. M., Field Theory, the Renormalization Group, and Critical Phenomena, World Scientific, 2005.

11. Creswick R. J. et al. Introduction to Renormalization Group Methods in Physics, John Wiley & Sons, 1992.